Will GEO Work? - An Economist View

A. Smirnov [a,b,*], M. Obersteiner [a]
[a] International Institute for Applied Systems Analysis (IIASA), Laxenburg, A-2361 Austria – (oberstei, smirnov)@iiasa.ac.at
[b] Department of Optimal Control, Faculty of Computational Mathematics and Cybernetics,
Moscow State University, Moscow 119991, Russia

Abstract – In this paper we use a game-theoretical approach to model a global partnership in building global earth observation system. Our analysis of possible equilibrium solutions shows that only in the case of similar economies we will observe cooperation behavior (when all invest into global system) and otherwise we will observe free-riding. However uncertainty in environmental risks valuing can provide a strong incentive for free-riders to cooperate.

Keywords: economic growth, uncertainty, natural disasters, preventive measures, game theory, self-enforcing cooperation.

1. INTRODUCTION

The emergence of a global partnership on earth observations will crucially drive the configuration of future observing systems and consequently shape how socio-economic benefits are generated. In this paper we use a game-theoretical approach to model cooperation on building global earth observation system. We consider societies whose economies are subject to shocks mimicking major natural disasters. Economies operate optimally and lead to the best possible expected value for the social welfares in the future. In order to increase its welfare even more society can make a decision to invest into a global system which lowers the risk of disasters. We start our investigation from a single-society case and show conditions under which benefits of such investment can be reaped. The propensity to invest increases with economic affluence and degree of vulnerability to natural disasters. We show that for poor and/or less vulnerable countries it is better to forbear from investment. In the situation of multiple societies a strategic gaming situation emerges motivated by the fact that every society will benefit from a global system regardless of whether they invested or not. Our analysis of possible equilibrium solutions shows that similar to the formation of trading blocks (e.g. EU, NAFTA) only in the case of similar societies we will observe cooperation behavior (when all invest) and otherwise we will observe free-riding. However uncertainty in environmental risks valuing can provide a strong incentive for free-riders to cooperate.

2. MODEL

We consider a stylized neoclassical model of the development of an economy affected by random natural hazards; the latter are treated as suggested in (Chladna et al, 2006).
Let us consider an economy operating over an infinite sequence of time periods, $i = 0,1,\dots$ In each period i the economy is characterized by the size of its capital stock, K_i, and the size of its production output, $Y_i = \alpha K_i$; here α is a given positive coefficient (we use the simplest one-factor Cobb–Douglas production function). In period 0 the initial size of the capital stock, $K_0 > 0$, is given. The economy is affected by natural hazards occurring randomly. In order to reduce the negative impact of natural hazards in periods 1,2,…, in period 0 part $z \in [0, Y_0)$ of the initial production output $Y_0 = \alpha K_0$ is invested in the development of prevention measures (building global earth observation systems allowing the economy to mitigate the future losses caused by natural hazards). The rest of the initial production output is divided between capital investment, I_0, and consumption, C_0

$$I_0 = s_0(Y_0 - z) = s_0(\alpha K_0 - z),$$
$$C_0 = (1 - s_0)(Y_0 - z) = (1 - s_0)(\alpha K_0 - z), \quad s_0 \in [0,1)$$

In period 0 an extreme event (a natural catastrophe) can occur with probability q_0; as a result the capital stock loses its fraction $d \in (0,1)$. Introducing a capital depreciation rate $\delta \in (0,1)$, we find that in period 1 the size of the capital stock is given by

$$K_1 = (K_0 + I_0 - \delta K_0)\zeta_0,$$

where ζ_0 is a random variable taking value $1 - d$ with probability q_0 and value 1 with probability $1 - q_0$. In each period $i = 1,2,\dots$ the capital stock K_i is divided between capital investment, I_i, and consumption, C_i

$$I_i = s_i Y_i = s_i \alpha K_i, \qquad C_i = (1 - s_i)Y_i = (1 - s_i)\alpha K_i,$$

here $s_i \in [0,1)$ is the savings rate of capital. In period i an extreme event occurs with probability q, causing the loss of fraction d of the capital stock. Accordingly,

$$K_{i+1} = (K_i + I_i - \delta K_i)\zeta_i,$$

where ζ_i is a random variable taking value $1 - d$ with probability q and value 1 with probability $1 - q$. Probability q endogenously depends on the preventive measures introduced in period 0, namely, we suppose that q is negatively related to the size of investment, z, more specifically, we set

$$q = \frac{q_0}{1 + \kappa z}, \tag{1}$$

where κ kappa is a given positive coefficient characterizing the efficiency of investment.

---

* Corresponding author.
** This research was performed in the framework of the EC project GEO-BENE (www.geo-bene.eu), led by the International Institute for Applied Systems Analysis (IIASA).

Social planner chooses economy's control parameters s_i, $i = 0,1,\ldots$, in order to maximize the economy's utility, expected value of the social welfare (discounted consumption),

$$W(z) = \max_{s_i} E\left(\sum_{i=0}^{\infty} (1+\rho)^{-1} \log C_i\right),$$

here ρ is a given positive discount rate.

The optimal solution for the model can be found analytically using dynamic programming method. The following proposition gives us the optimal savings rate and welfare (see (Kryazhimskiy et al., 2008) for details).

Proposition 1. *For every* $z \in [0, \alpha K_0)$, *the optimization problem has the unique solution*

$$s_0 = \begin{cases} \frac{\alpha K_0 - z - \rho(1-\delta)K_0}{(\alpha K_0 - z)(1+\rho)} & \text{if } z < (\alpha + \rho\delta - \rho)K_0, \\ 0 & \text{otherwise} \end{cases}$$

$$s_i = \frac{\alpha - \rho(1-\delta)}{\alpha(1+\rho)} \quad (i = 1,2,\ldots),$$

$$W(z) = \log(1 - s_0) + \tfrac{1}{\rho}\log\big((1-\delta)K_0 + s_0(\alpha K_0 - z)\big) + \log(\alpha K_0 - z) + \tfrac{1}{\rho}\log\rho + \tfrac{1+\rho}{\rho^2}\log\rho\left(\tfrac{\alpha+1-\delta}{1+\rho}\right)$$

Thus we know now how to run optimally the economy to achieve the perfect (expected) welfare. Could we make it better somehow? Yes, we can! We can raise a problem of optimal investment *z* in prevention measures

$$\text{Maximize } W(z) \text{ over all } z \in [0, \alpha K_0). \qquad (2)$$

Proposition 2. *The problem* (2) *has the unique solution* z^*:
if

$$\kappa K_0 |q_0 \log(1-d)| \le \frac{\rho(1+\rho)}{1+\alpha-\delta}, \qquad (3)$$

then $z^* = 0$,
if the inequality opposite to (3) holds then z^* *is positive and given by*

$$z^* = \begin{cases} z_1^* & \text{if } \kappa K_0 |q_0 \log(1-d)| \le \frac{\rho(1+\kappa(\alpha+\delta\rho-\rho)K_0)^2}{1+\alpha-\delta}, \\ z_2^* & \text{otherwise}, \end{cases}$$

where

$$z_1^* = \frac{(q_0\log(1-d))^{1/2}(q_0\log(1-d) - 4\rho(1+\rho)(1+(1+\alpha-\delta)\kappa K_0))^{1/2}}{2\rho\kappa(1+\rho)} - \frac{1}{\kappa} + \frac{q_0\log(1-d)}{2\rho\kappa(1+\rho)},$$

$$z_2^* = \frac{(q_0\log(1-d))^{1/2}(q_0\log(1-d) - 4\rho^2(\alpha\kappa K_0+1))^{1/2}}{2\rho^2\kappa} - \frac{1}{\kappa} + \frac{q_0\log(1-d)}{2\rho^2\kappa}.$$

Consider, in more detail, the cases where the optimal investment in the prevention measures, z^*, is zero and positive, respectively. The right hand side of (3) determining the case $z^* = 0$ involves parameters characterizing the economy's dynamics only, whereas its left hand side, $\kappa K_0 |q_0 \log(1-d)|$, is clearly related to natural hazards. Indeed, the product κK_0 characterizes the economy's ability to cope with natural hazards in period 0 (recall that κ is the efficiency of investment in the prevention measures, and K_0 is the size of the initial capital stock); and the product $|q_0 \log(1-d)|$ acts as a measure of danger caused by natural hazards; it grows as q_0 (the initial probability of natural hazards) and *d* (the fractional size of losses due to natural hazards) grow. Inequality (3) tells us, roughly, that either the economy has a low ability to cope with natural hazards, or the measure of danger, caused by natural hazards is not high enough. In this situation the economy refrains from investing in the prevention measures in period 0: $z^* = 0$. Conversely, the inequality opposite to (3) tells us, roughly, that either the economy has a high ability to cope with natural hazards, or the measure of danger, caused by natural hazards is quite high. In this situation the economy invests a positive resource in the prevention measures in period 0: $z^* > 0$.

3. INVESTMENT GAME

Now we consider the case of two independent economies both under the threat of natural disasters. Each of the economies can make an investment in common prevention measures aimed at mitigating the impact of natural hazards on both economies. We suppose that each economy is subject the same dynamics as in the previous section but with its own set of parameters. We only need to modify the rule (1) to introduce a joint effect of prevention measures; namely we assume that *q*, the probability of the occurrence of natural hazards after the implementation of the prevention measures, is negatively related to the economies' investments, z^1 and z^2, more specifically, we set

$$q = \frac{q_0}{1 + \kappa^1 z^1 + \kappa^2 z^2}, \qquad (4)$$

where κ^1, κ^2 characterize the efficiency of investment of economies. As in the previous section each economy is maximizing its utility by choosing the value for savings rate at each period $i = 0,1,\ldots$ To reflect the indirect (thru (4)) influence of the investment into prevention measures made by one economy to the welfare of other economy we will use the notations $W_1(z^1, z^2)$ and $W_2(z^1, z^2)$ instead of $W_1(z^1)$ and $W_2(z^2)$. Optimal values for savings rate and utility in each economy follow the Proposition 1 (with some obvious changes).

Investment game appears as soon as we raise a problem of finding a pair of values (z^1, z^2) which maximizes the welfares of both economies

$$\text{Maximize } W_1(z^1, z^2) \text{ over all } z^1 \in [0, \alpha^1 K_0^1).$$
$$\text{Maximize } W_2(z^1, z^2) \text{ over all } z^2 \in [0, \alpha^2 K_0^2).$$

Our goal is to characterize the Nash equilibrium in such a game. To do this we solve each maximization problem independently and construct so called functions of best reply $z^{*1}(z^2)$ and $z^{*2}(z^1)$. More specifically, e.g., $z^{*1}(z^2)$ is the optimal investment of the first economy in the case where the second economy invested z^2. These functions can be described in an analytical way very similar to Proposition 2 but we will not show them here to avoid very big formulas (see (Kryazhimskiy et al., 2008) for details). We only mention one important case (similar to (3)): if

$$\kappa^1 K_0^1 |q_0 \log(1-d)| \le \frac{\rho(1+\rho)(1+\kappa^2 z^2)^2}{1+\alpha^1-\delta^1}, \qquad (5)$$

then $z^{*1}(z^2) = 0$.

What is also important to note here is that functions $z^{*1}(z^2)$ and $z^{*2}(z^1)$ are turn to be almost (piecewise) linear decreasing functions of their arguments.

Proposition 3. *Investment game problem always has a unique Nash equilibrium solution (*z^{*1}, z^{*2}*) which is the solution of the following system of equations*

$$\begin{cases} z^{*1}(z^2) = z^1, \\ z^{*2}(z^1) = z^2. \end{cases} \qquad (6)$$

All equilibrium solutions could be classified into three following cases: 1) Both economies refrain from investment into preventing measures; 2) Only one economy invests; 3) Both economies invest. Cases 1 and 2 we call non-cooperative behavior, Case 3 is cooperative. It is quite obvious that if an economy doesn't invest in a single economy framework (see (3)) then it doesn't invest in the game framework as well. However even for the economy with the initial propensity to invest this propensity vanishes at some critical level of the investment of the other economy (see (5)). Further analysis reveals that most frequently we get a non-cooperative outcome in the investment game.

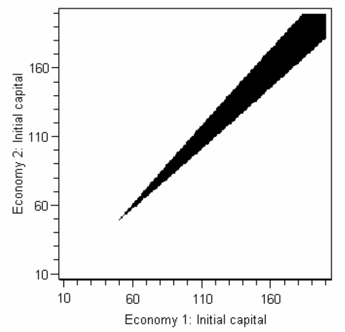

Figure 1.

Figure 1 shows the example of "cooperation zone" on the plane of model parameters K_0^1 and K_0^2 (initial states of the economies) given that all other parameters are fixed and equal in both economies. In fact this numerical example represents a general feature of the game: economies should be quite similar to be cooperative in investments into preventive measures.

4. ROLE OF UNCERTAINTY

As we have just seen "poor" and "rich" economies are hardly to cooperate in the investment game. However, such a conclusion is valid only in the context of perfect knowledge about model's parameters. And as long as we are talking about economical parameters (capital stock, depreciation rate, etc.) this can be true. But we unlikely know precise values for the probability of natural disasters (parameter q_0) and their impact on capital stock (parameter d). Uncertainty analysis of equilibrium solutions generated by system (6) shows that for the some of previously non-cooperative economies there will appear additional cooperative solutions. Figure 2 shows that 10% uncertainty in the probability q_0 of occurring of natural disaster leads to the increasing of "cooperation zone" of Figure 1 more than twice. Grey area on the figure describes the economies where cooperation becomes an option.

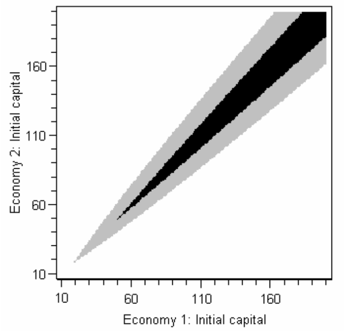

Figure 2.

However to "compensate" these wider cooperation cases we lose the uniqueness of equilibrium solution in the game (cf. Proposition 3). And this is a serious loss because (without any special assumptions concerning uncertainty itself) we get an infinite set of possible cooperative equilibriums.

5. CONCLUSIONS

We used a game-theoretical approach to model cooperation of independent economies on building global system aimed on mitigation of future economical losses caused by natural disasters. Our analysis of global partnership shows that partnership is naturally emerge among similar economies but uncertainty in environmental risks valuing provides a strong incentive for cooperation for broader spectrum of economies.

REFERENCES

Z. Chladna, E. Moltchanova, and M. Obersteiner, "Prevention of Surprise", in: S. Albeverio, V. Jentsch, H. Kantz (Eds.), Extreme Events in Nature and Society, Springer, vol. 352, pp. 295–318, 2006.

A. Kryazhimskiy, M. Obersteiner, and A. Smirnov, "Infinite-horizon dynamic programming and application to management", Appl. Math. Comput., 205, pp. 609–620, (doi:10.1016/j.amc.2008.05.042), 2008.